AFCRC-TN-59-156
ASTIA DOCUMENT NO. AD 212464

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NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

RESEARCH REPORT No. EM-128

Excitation and Propagation of Surface Waves

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Contract No. AF19(604)1717

FEBRUARY, 1959

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The research reported in this document has been sponsored by the Electronics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, and by a grant from the American Petroleum Institute. The publication of this report does not necessarily constitute approval by the Air Force of the findings or conclusions contained herein.

Contract No. AF 19(604)1717

February, 1959

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Abstract

A geometrical theory is developed for the analysis of surface wave excitation and propagation. The surfaces along which the surface waves propagate may be either curved or flat, and may have either constant or variable properties. The theory is based on the concept of a complex or imaginary ray. The excitation coefficient which enters the theory is determined from the solution of a canonical problem - that of a line source over an impedance plane. Then the theory is applied to the surface wave excited by a line source, on a wedge with variable surface impedance. The result agrees precisely with the asymptotic form of the exact solution. Another application is made to the surface wave excited on a cylinder by a line source. The result also agrees with the exact solution.

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1. Introduction

A surface wave is a wave which travels along a surface with its field confined to the neighborhood of the surface. It differs from other waves with this property in that it can exist by itself on a flat surface in a homogeneous medium. The evanescent waves produced by total reflection are also confined to the neighborhood of a surface, but they cannot exist without the incident field. On the other hand, the diffracted waves associated with the usual surface rays can exist by themselves on curved surfaces, but not on flat ones. A surface wave is characterized by a complex propagation constant $k_1 = \omega/c_1$, where ω is the angular frequency of the field and c_1 is the complex velocity. Either k_1 or c_1 determines the phase velocity and the attenuation rate of the wave along the surface. We shall assume that k_1 and c_1 are known and investigate the manner in which such waves are excited, how they propagate and the rate at which their field diminishes with distance from the surface. We shall do this by constructing a geometrical theory, analogous to geometrical optics and to the geometrical theory of diffraction [1], which will permit us to visualize the excitation process and to calculate the amplitude of the surface wave in rather general cases.

In the next three sections we formulate our theory. In the fifth and sixth sections we determine the excitation and radiation coefficients which enter the theory. This is done for scalar fields which satisfy the reduced wave equation and satisfy an impedance condition on the surface. The problem of designing the source to maximize the surface wave amplitude is also treated. Then the method is used to determine the surface wave which a line source excites on a wedge with variable surface impedance. The result is found to coincide with the high frequency asymptotic form of the exact solution of this problem. Finally the surface wave on a circular cylinder is determined by our method and also found to coincide with the asymptotic form of the exact solution.

2. Complex rays

Rays in space can be defined in terms of the propagation velocity $c(\vec{x})$ by Fermat's principle or by the differential equations which follow from it. Complex rays can be defined by applying Fermat's principle to complex curves, or by considering complex solutions of the ray equations. In either case it is necessary that $c(\vec{x})$ be defined for complex values of $\vec{x}$, which is the case if $c(\vec{x})$ is an analytic function of $\vec{x}$. We can also define complex surface rays by applying Fermat's principle to complex curves on the surface S or by considering complex solutions of the equations for surface rays. In this case it is necessary that both the velocity $c_1(\vec{x})$ and the surface S be defined for complex $\vec{x}$.

Next, by means of Fermat's principle, we may define a ray joining two points Q and P in space and having an arc on S. It follows from Fermat's principle that such a ray consists of a complex space ray from Q to some point Q_1 on S, a complex surface ray from Q_1 to another point P_1 on S, and a complex space ray from P_1 to P. In addition it follows that at both Q_1 and P_1 the unit tangents I and T to the space and surface rays respectively must satisfy the condition

$$c^{-1}I + c_1^{-1}T = (c^{-1}I \cdot N + c_1^{-1}T \cdot N)N. \tag{1}$$

Here N denotes the unit normal to S at Q_1 or P_1. The vectors are oriented such that -I and T both point in the direction of increasing arclength along the ray. See Figure 1. Equation (1) is equivalent to the geometrical conditions that I

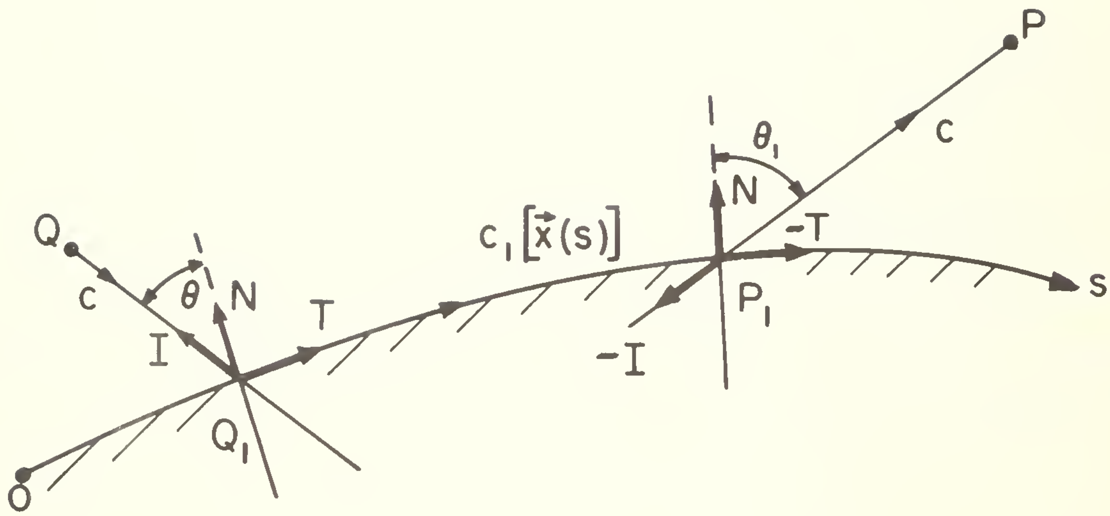

FIGURE I

The complex ray from Q reaches the surface at the complex point Q_1 where it produces a complex surface ray. This ray travels along the surface with the velocity c_1 and sheds complex space rays. The ray which leaves the surface at P_1 passes through P. The angles θ and θ_1 are determined by Snell's law (1).

and T be coplanar with N and lie on opposite sides of N, and that the angle θ between I and N satisfy Snell's law

$$\sin \theta = \frac{c}{c_1} \quad . \tag{2}$$

The preceding considerations enable us to determine all the ray paths which contain arcs on S along which surface wave propagation occurs.

3. Amplitude and phase

The field on any ray is composed of a phase and an amplitude. On a surface ray from Q_1 to P_1 the complex phase change $\phi\,(Q_1, P_1)$ is the following integral with respect to arc length s

$$\phi(Q_1, P_1) = \omega \int_{Q_1}^{P_1} \frac{ds}{c_1\left[\vec{x}(s)\right]} = \int_{Q_1}^{P_1} k_1\left[\vec{x}(s)\right] ds \ . \tag{3}$$

On a space ray

$$\phi(P_1, P) = \omega \int_{P_1}^{P} \frac{ds}{c\left[\vec{x}(s)\right]} = \int_{P_1}^{P} k\left[\vec{x}(s)\right] ds \ . \tag{4}$$

The amplitude variation along any ray depends upon the family of rays in which the ray under consideration is continued. For a complex ray the amplitudes $A_1(Q_1)$ and $A_1(P_1)$, which are vectors for vector fields, are related by

$$A_1(P_1) = A_1(Q_1)\left[\frac{d\sigma(Q_1)c_1(P_1)}{d\sigma(P_1)c_1(Q_1)}\right]^{1/2} \quad . \tag{5}$$

In (5) $d\sigma(Q_1)/d\sigma(P_1)$ denotes the ratio of the width at Q_1 to that at P_1 of an infinitesimal band of complex surface containing the ray from Q_1 to P_1. The analogue of (5) for the space ray from P_1 to P is

$$A(P) = A(P_1)\left[\frac{d\tau(P_1)c(P)}{d\tau(P)c(P_1)}\right]^{1/2} . \tag{6}$$

In (6) $d\tau(P_1)/d\tau(P)$ is the ratio of the cross sectional area at P_1 to that at P of an infinitesimal tube of complex rays containing the ray from P_1 to P. A similar relation holds on any space ray.

The amplitude on the surface ray at Q_1 is proportional to that on the space ray at Q_1 so we have

$$A_1(Q_1) = E(Q_1)A(Q_1) . \tag{7}$$

The proportionality factor $E(Q_1)$ in (7) is an excitation coefficient. In the case of vector fields, it is a matrix. Similarly, the amplitude on the space ray at P_1 is proportional to that on the surface ray there so we have

$$A(P_1) = R(P_1)A_1(P_1) . \tag{8}$$

The proportionality factor or matrix, $R(P_1)$ in (8) is a radiation coefficient. Upon combining the above results, we find that the field u(P) at P is given by

$$u(P) = R(P_1)E(Q_1)A(Q_1)\left[\frac{d\sigma(Q_1)c_1(P_1)d\tau(P_1)c(P)}{d\sigma(P_1)c_1(Q_1)d\tau(P)c(P_1)}\right]^{1/2} e^{i\left[\phi(Q_1)+\phi_1(Q_1,P_1)+\phi(P_1,P)\right]} . \tag{9}$$

Equation (9) expresses the field at P on a ray which has traveled along an arc of S as a complex surface ray associated with surface wave propagation.

It involves the incident field $A(Q_1)e^{i\phi(Q_1)}$, various geometrical quantities, and the excitation and radiation coefficients.

We shall now restrict our considerations to the two dimensional case. In addition we shall assume that c is constant so that the space rays are straight lines. Due to the two dimensionality, the ratio $d\sigma(Q_1)/d\sigma(P_1) = 1$. Because the rays are straight we have $\phi(P_1,P) = kd(P_1,P)$ where $d(P_1,P)$ is the complex distance from P_1 to P. In addition we find that

$$(10) \qquad \frac{d\tau(P_1)}{d\tau(P)} = \left[1+(\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\right]^{-1}_{P_1} .$$

In (10) $\mathcal{K}(P_1)$ denotes the curvature of S at P_1, $\theta(P_1)$ is the solution of (2) at P_1, and $\dot{\theta}$ is the derivative of θ with respect to arclength along the ray on S, evaluated at P_1. With these simplifications, (9) becomes

$$(11) \quad u(P) = R(P_1)E(Q_1)A(Q_1)\left[\frac{c_1(P_1)}{c_1(Q_1)}\right]^{1/2}\left[1+(\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\right]^{-1/2}_{P_1} e^{i\left[\phi(Q_1)+\phi_1(Q_1,P_1)+kd(P_1,P)\right.}$$

4. The reciprocity principle

To determine the coefficients $R(P_1)$ and $E(Q_1)$ we will now apply (11) to the case in which the field is produced by an isotropic line source, represented by the point Q in two dimensions. Then the incident field is given by

$$(12) \qquad A(Q_1)e^{i\phi(Q_1)} = A_o\left[d(Q,Q_1)\right]^{-1/2}e^{ikd(Q,Q_1)} .$$

In (12) A_o is a constant characteristic of the source, with the dimensions of $(\text{length})^{1/2}$ times those of u. When use is made of (12), (11) becomes

$$u(P) = R(P_1)E(Q_1)A_o\left[d(Q,Q_1)\right]^{-1/2}\left[1+(\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\right]_{P_1}^{-1/2} \tag{13}$$

$$\left[\frac{c_1(P_1)}{c_1(Q_1)}\right]^{1/2} e^{i\left[kd(Q,Q_1)+\phi(Q_1,P_1)+kd(P_1,P)\right]} \quad .$$

We now make use of the reciprocity principle which requires that a source at P must produce at Q the same field as an equal source at Q would produce at P. This implies that the right side of (13) must be unchanged when P is interchanged with Q and P_1 with Q_1. From this constancy we obtain the condition

$$R(P_1)E(Q_1)\left[d(Q,Q_1)\right]^{-1/2}\left[1+(\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\right]_{P_1}^{-1/2}\left[\frac{c_1(P_1)}{c_1(Q_1)}\right]^{1/2} \tag{14}$$

$$= R(Q_1)E(P_1)\left[d(P,P_1)\right]^{-1/2}\left[1+(\mathcal{K}+\dot{\theta})d(Q_1,Q)\sec\theta\right]_{Q_1}^{-1/2}\left[\frac{c_1(Q_1)}{c_1(P_1)}\right]^{1/2}$$

Equation (14) enables us to express the excitation coefficient E in terms of the radiation coefficient R by the relation

$$E(Q_1) = a^2R(Q_1)\left[\mathcal{K}_i+(\mathcal{K}+\dot{\theta})\sec\theta\right]_{Q_1}^{-1/2} c_1(Q_1) \quad . \tag{15}$$

Here a^2 is a constant with dimensions $(\text{time})\times(\text{length})^{-3/2}$. From (15) we see that $E(Q_1)$ depends upon $\mathcal{K}_i = 1/d(Q,Q_1)$, the curvature of the incident wavefront at Q_1, in addition to properties of the surface at Q_1. It will be convenient to introduce $R'(Q_1) = a\left[c_1(Q_1)\right]^{1/2}R(Q_1)$ in order to avoid carrying along the factor a.

If we now use (15) to eliminate $E(Q_1)$ from (13), we obtain

$$(16)\quad u(P) = R'(P_1)R'(Q_1)A_o\left[1+(\mathcal{K}+\mathring{\theta})d(Q_1,Q)\sec\theta\right]_{Q_1}^{-1/2}\left[1+(\mathcal{K}+\mathring{\theta})d(P_1,P)\sec\theta\right]_{P_1}^{-1/2}$$

$$\times\ e^{i\left[kd(Q,Q_1)+\phi(Q_1,P_1)+kd(P_1,P)\right]}\quad .$$

This expression for $u(P)$ applies in the two dimensional case when c is constant and the field is produced by a line source at Q. For a more general incident field we must use (15) in (11).

If the surface S is a plane and if c_1 is constant then ρ is infinite, $\mathring{\theta}$ is zero, $R'(P_1) = R'(Q_1) = R'$, $\phi(Q_1,P_1) = k_1 d(Q_1,P_1)$ and (16) becomes

$$(17)\qquad u(P) = R'^2 A_o e^{i\left[kd(Q,Q_1)+k_1d(Q_1,P_1)+kd(P_1,P)\right]}\quad .$$

5. <u>Determination of the coefficients</u>

To complete the prescription for calculating the field, we must determine the radiation coefficient R'. Since R is dimensionless, R' has the dimensions of $(\text{length})^{-1/4}$. Therefore it will be more convenient to consider the dimensionless product $k^{-1/4}R'(P_1)$. This product must depend upon the type of field under consideration. We shall assume that it depends upon $\theta(P_1)$ and possibly upon other physical constants of the medium and the surface at P_1, but not upon any geometrical properties of the surface at P_1. Therefore it can be found by considering the special problem of a line source in the presence of a uniform plane surface in a homogeneous medium. By comparing the exact solution of such a problem with (17), after expanding the exact solution for large ω (or large k and k_1), R' can be determined. In order to facilitate such a comparison, we will

now calculate the distances which appear in (17) in terms of the coordinates of P and Q.

Let us assume that the surface occupies the plane $y = 0$, that the source Q is located at $(0,y_o)$, and that the point P is at (x,y) with $x > 0$. The ray from Q to Q_1 is a straight line which makes with the y-axis the angle θ, determined by (2). Similarly the ray from P_1 to P is a straight line making the same angle with the y-axis. Therefore the coordinates of Q_1 are $(y_o\tan\theta,0)$ and those of P_1 are $(x-y\tan\theta,0)$. See Figure 2. Then we find that $d(Q,Q_1) = y_o\sec\theta$, $d(P_1P) = y\sec\theta$ and $d(Q_1,P_1) = x-(y_o+y)\tan\theta$. When these distances are inserted into (17), it becomes

$$(18) \qquad u(x,y) = R'^2 A_o e^{ik\left[|x|\sin\theta+(y_o+y)\cos\theta\right]} \quad .$$

Here x has been replaced by $|x|$ in order that this result remain valid for $x < 0$. In obtaining (18) from (17) we used the relation $k/k_1 = \sin\theta$, which follows from (2).

In the next section we shall obtain an exact solution of the problem just treated for a scalar field which satisfies the reduced wave equation

$$(19) \qquad (\nabla^2+k^2)u = \delta(x)\delta(y-y_o) \quad , \qquad y > 0 \ .$$

On the plane $y = 0$, u will satisfy the impedance boundary condition

$$(20) \qquad u_y = -ikZu \ .$$

In (20) the complex constant Z is the surface impedance. In order for the surface to support a surface wave, it is necessary that Z satisfy the conditions

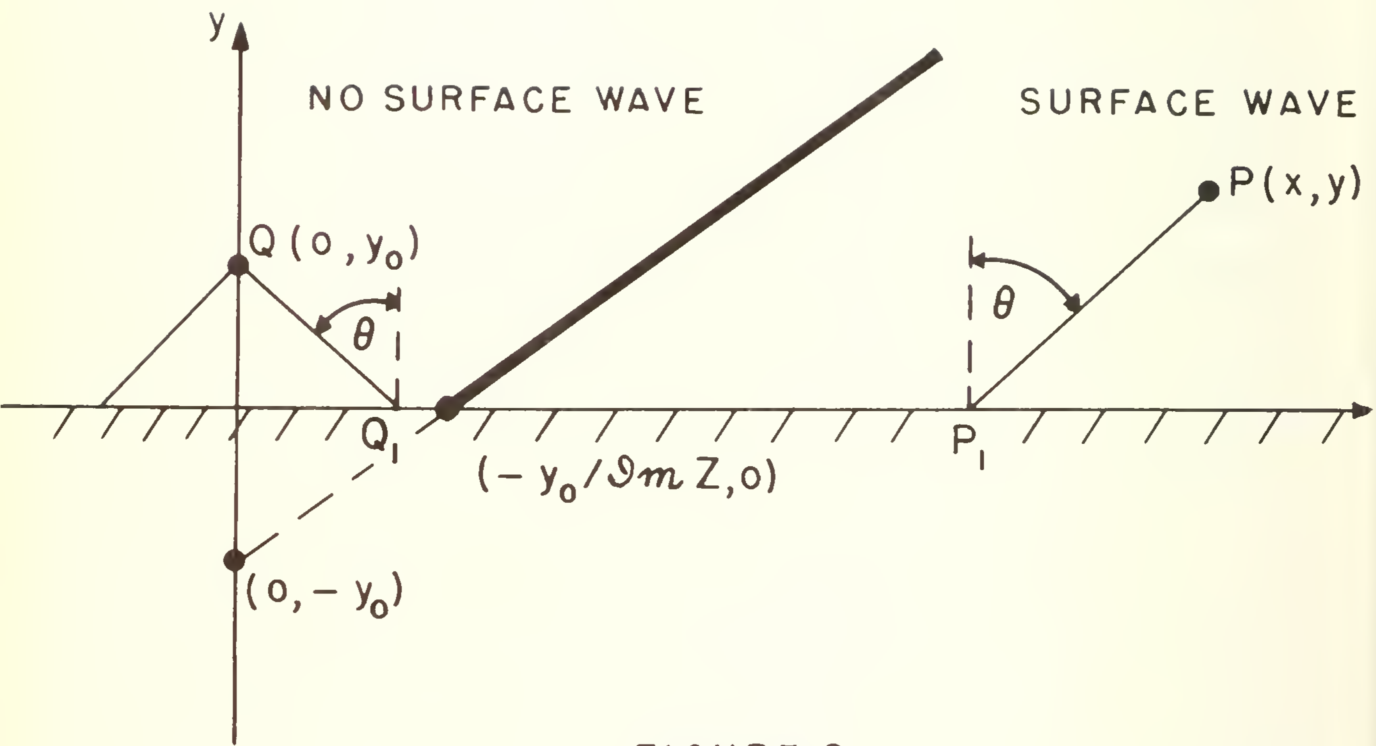

FIGURE 2

The surface wave produced by a line source at Q above a plane of surface impedance Z. The figure shows a ray from Q which makes the complex angle Θ with the normal to the plane. The angle Θ is determined by equation (22). This ray gives rise to a complex surface ray which sheds complex space rays. One such ray is shown leaving the surface at P_1 and passing through P. These rays cover the region to the right of the real line $(y + y_o)x^{-1} = -\mathrm{Im}Z$ provided $\mathrm{Re}\ Z = 0$. The figure is symmetric about the y axis.

(21) $$\mathrm{Re}Z \geq 0 \, , \quad \mathrm{Im}Z < 0 \, .$$

If (18) is inserted into (20) it yields

(22) $$\cos\theta = -Z \, .$$

From (22) and (2) the propagation c_1 is determined by the surface impedance through the relation

(23) $$c_1 = c(1-Z^2)^{-1/2} \, .$$

When the outgoing solution of (19) and (20) is expanded asymptotically for large values of k, it contains a surface wave of the form (18) with θ given by (22). This surface wave becomes identical with (18) if in (18) we set

(24) $$A_o = (8\pi k)^{-1/2} \, e^{-i\pi/2}$$

(25) $$R' = (8\pi k)^{1/4} \, e^{i\pi/4} \, (\cot\theta)^{1/2} \, .$$

This equation determines the radiation coefficient.

Let us now insert (24) and (25) into (16). This yields the final result for the surface wave excited on an impedance surface by an isotropic line source of amplitude A_o in a homogeneous medium. It is

(26) $$u(P) = \Big[\tan\theta+(\mathcal{K}+\dot{\theta})d(Q_1,Q)\sec\theta\tan\theta\Big]_{Q_1}^{-1/2}\Big[\tan\theta+(\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\tan\theta\Big]_{P_1}^{-1/2}$$

$$\times \; e^{i\left[kd(Q_1,Q_1)+\phi(Q_1,P_1)+kd(P_1,P)\right]}$$

If the line source is not isotropic we may write its amplitude as $A_o f(\theta)$. Then the right side of (26) must be multiplied by $f(\theta)$. Here $f(\theta)$ represents the radiation pattern of the source. It is proportional to the amplitude on a ray which makes the angle θ with the normal to the surface S. Usually the pattern is expressed as a function $F(\Psi)$ of the angle Ψ which the ray makes with a fixed direction. By determining Ψ as a function of θ from the shape of S and the location of Q, we then have $f(\theta) = F[\Psi(\theta)]$. To maximize the amplitude of the surface wave it is necessary to maximize the pattern $f(\theta)$ in the complex direction θ of the ray which excites the surface wave. This direction is determined by (2) or (22). If S is a plane and if Ψ is measured from the line through Q normal to S, then $\Psi = \theta$ and therefore $f(\theta) = F(\theta)$.

If the incident field is arbitrary, the surface wave is given by (11), (15) and (25). Upon combining these equations we obtain

$$(27) \quad u(P) = (8\pi k)^{1/2} u_i(Q_1)\left[\mathcal{K}_i \tan\theta + (\mathcal{K}+\dot{\theta})\sec\theta\tan\theta\right]_{Q_1}^{-1/2}\left[\tan\theta + (\mathcal{K}+\dot{\theta})d(P_1,P)\sec\theta\tan\theta\right]_{P_1}^{-1/2}$$

$$\times \; e^{i\left[\pi/2+\phi_1(Q_1,P_1)+kd(P_1,P)\right]} \; .$$

Here $u_i(Q) = A(Q)e^{i\phi(Q_1)}$ denotes the incident field at Q_1.

6. Surface wave on an impedance plane

We shall now solve the boundary value problem (19) and (20), also requiring u to satisfy the radiation condition. The solution is

$$(28) \quad u(x,y) = -\frac{i}{4} H_o^{(1)}(kr) + \frac{i}{4\pi}\int_C \frac{\sin\phi - Z}{\sin\phi + Z}\, e^{ik\left[x\cos\phi + (y+y_o)\sin\phi\right]} d\phi \; .$$

In (28) r denotes the distance from $(0,y_o)$ and C is a contour in the complex plane. It runs from $\pi - i\infty$ to π along the line $\text{Re}\phi = \pi$, then from π to 0 along the line $\text{Im}\phi = 0$ and finally from 0 to $i\infty$ along $\text{Re}\phi = 0$. See Figure 3.

To obtain the asymptotic form of u for large k, we can first use the asymptotic form of $H_o^{(1)}(kr)$ for large values of the argument. This yields the asymptotic form of the incident field. The asymptotic form of the reflected field is obtained by performing a saddle point evaluation of the integral in (28). The saddle point ϕ_o which lies in the interval $0 < \phi_o < \pi$ is given by

$$\phi_o = \tan^{-1} \frac{(y+y_o)}{|x|} \quad . \tag{29}$$

The appropriate contour through the saddle point is obtained by setting the imaginary part of the phase $x\cos\phi+(y+y_o)\sin\phi$ equal to its value at $\phi = \phi_o$. If we let $\phi = \alpha+i\beta$, we find in this way two possible contours

$$\alpha = \phi_o \pm \cos^{-1}(\text{sech}\beta) \; . \tag{30}$$

In order that the exponential in the integrand of (28) vanish at infinity on the contour, we must choose

$$\alpha = \phi_o+\cos^{-1}(\text{sech}\beta) \qquad \beta < 0 \tag{31}$$

$$\alpha = \phi_o-\cos^{-1}(\text{sech}\beta) \qquad \beta > 0 \tag{32}$$

With this choice of signs, the contour C can be deformed into the saddle contour. See Figure 3. Then the saddle point evaluation of the integral yields the asymptotic form of the reflected field.

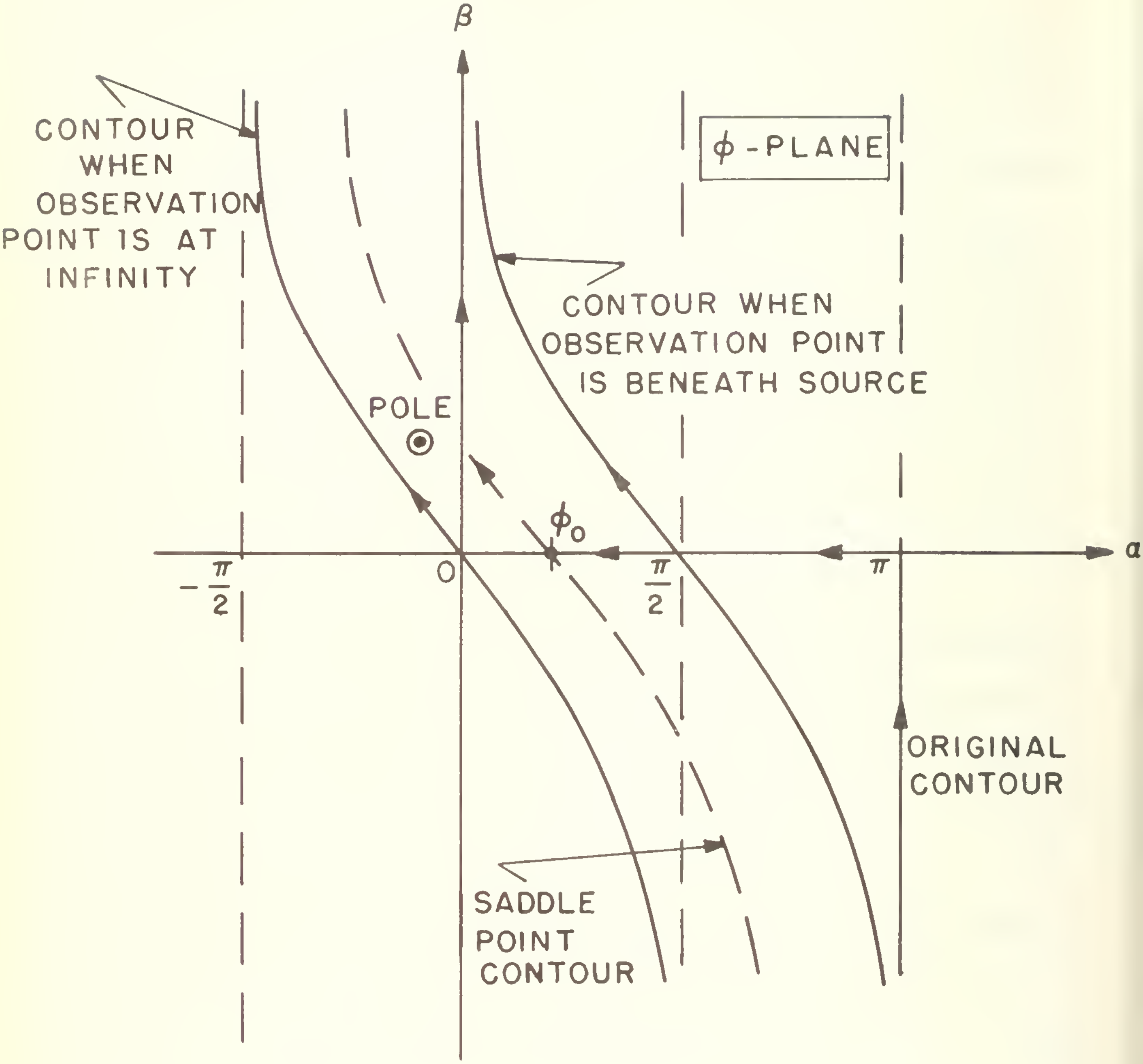

FIGURE 3

Saddle point contour used in the asymptotic evaluation of the reflected field for a line source above a plane with a constant surface impedance.

In the process of deforming C into the saddle contour it may cross the pole of the integrand which is at ϕ_1 given by

$$\sin\phi_1 = -Z \quad . \tag{33}$$

If this occurs, the residue of the integral at this pole must be added to the saddle point contribution to u. The residue is just the surface wave. It is easily found to be given by

$$u = \frac{Z}{\sqrt{1-Z^2}} e^{ik\left[|x| \sqrt{1-Z^2} - (y+y_o)Z\right]} \quad . \tag{34}$$

We have already stated that this coincides with (18) when A_o and R' are given by (24) and (25), and Θ by (22).

To determine when the surface wave (34) is present, let us consider the saddle contour which passes through the pole. Let ϕ^* denote the value of ϕ_o for this contour. Because Z satisfies (21), the pole lies in the upper half plane $\beta > 0$. Thus (32) yields for ϕ^* the result

$$\phi^* = \alpha_1 + \cos^{-1}(\operatorname{sech}\beta_1) \quad . \tag{35}$$

If $\phi_o < \phi^*$, the pole is crossed and the surface wave occurs, if $\phi_o > \phi^*$ it does not occur. From (29) we thus find that the surface wave occurs if and only if the following condition is satisfied

$$\frac{y+y_o}{|x|} < \tan\left[\alpha_1 + \cos^{-1}(\operatorname{sech}\beta_1)\right] \quad . \tag{36}$$

In (36) α_1 and β_1 are the real and imaginary parts of ϕ_1, the solution of (33). In the special case in which $\mathrm{Re}Z = 0$, (33) shows that $\alpha_1 = 0$. Then $\beta = -\sinh^{-1}(\mathrm{Im}Z)$ and (36) becomes

$$\frac{y+y_o}{|x|} < -\mathrm{Im}Z \quad . \tag{37}$$

See Figure 2.

7. Surface wave on a wedge with a non-uniform impedance

Let us now apply our method to determine the surface wave which a line source excites on a wedge with a non-uniform surface impedance. If the line source is parallel to the edge of the wedge, this problem is two dimensional. Then the point Q at which the source is located can be described by its polar coordinates (ρ',ϕ') and any other point P by (ρ,ϕ). The origin is at the tip of the wedge, the surfaces of which are at $\phi = 0$ and $\phi = \alpha$. See Figure 4. If the incident wave has the amplitude (24) then the surface wave which it produces on either surface of the wedge is given by (26). To determine this wave excited on $\phi = 0$ we assume that Z, the surface impedance of the wedge on $\phi = 0$, is proportional to ρ^{-1}. Thus

$$Z = Z_0\rho^{-1} \tag{38}$$

Here Z_0 is a constant which satisfies (21). Then from (23)

$$c_1 = \rho c(\rho^2-Z_0^2)^{-1/2} \quad . \tag{39}$$

To write (26) more explicitly we must evaluate the various geometrical quantities which appear in it. We shall first give them when P is to the right of Q, so that $\rho\cos\phi > \rho'\cos\phi'$. Then

$$d(Q,Q_1) = \rho'\sin\phi'\sec\theta' \tag{40}$$

$$d(P,P_1) = \rho\sin\phi\sec\theta \tag{41}$$

$$\rho(Q_1) = \rho'\cos\phi'+\rho'\sin\phi'\tan\theta' = \rho'\sec\theta'\cos(\phi'-\theta') \tag{42}$$

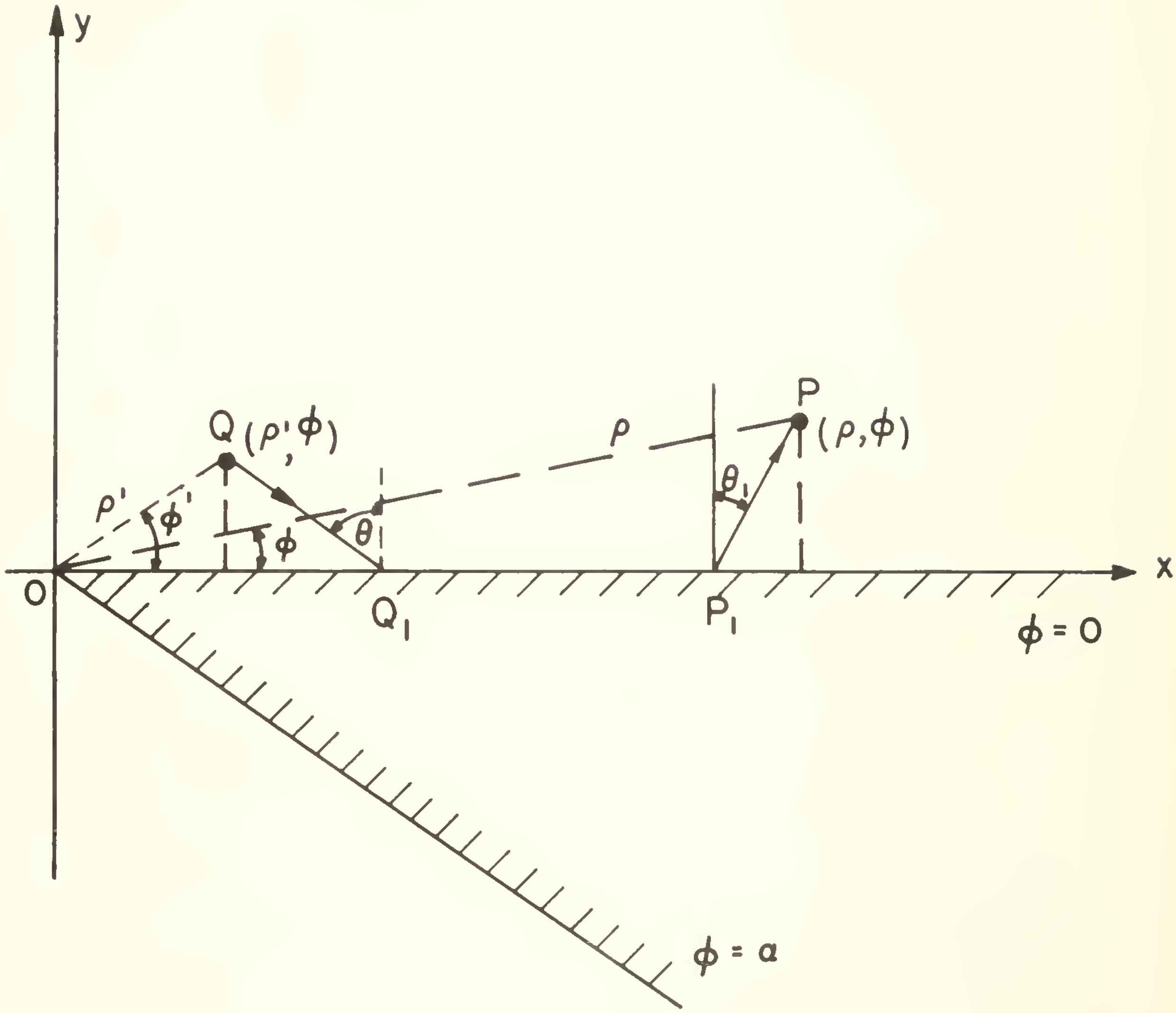

FIGURE 4

Excitation of a surface wave on a wedge by a line source at Q. The coordinates of Q are (ρ',ϕ'). A complex ray from Q excites a surface ray at the complex point Q_1. This ray travels along the surface shedding complex space rays. The ray shed at P_1 passes through P which has coordinates (ρ,ϕ). The wedge faces are at $\phi = 0$ and $\phi = \alpha$. The complex angles θ and θ_1 are determined by equation (22) in which Z is given by equation (38).

(43) $$\rho(P_1) = \rho\cos\phi - \rho\sin\phi\tan\theta = \rho\sec\theta\cos(\phi+\theta).$$

From (22) and (38) we have

(44) $$\rho(Q_1) = -Z_o\sec\theta'$$

(45) $$\rho(P_1) = -Z_o\sec\theta .$$

Now from (42) and (44) we determine θ', and from (43) and (45) we find θ.

(46) $$\theta' = \phi' + \cos^{-1}(-Z_o/\rho')$$

(47) $$\theta = -\phi + \cos^{-1}(-Z_o/\rho) .$$

From (3) and (39) we obtain

(48) $$\phi_1(Q_1,P_1) = k\int_{\rho(Q_1)}^{\rho(P_1)} \rho^{-1}(\rho^2-Z_o^2)^{1/2}d\rho$$

$$= k\left[\rho\cos(\phi+\theta)\tan\theta - \rho'\cos(\phi'-\theta')\tan\theta' + Z_o(\theta'-\theta)\right] .$$

Upon combining (40), (41) and (48) and simplifying, we obtain for the phase in (26)

(49) $$kd(Q,Q_1)+\phi_1(Q_1,P_1)+kd(P_1,P)$$

$$= k\left[\rho(1-Z_o^2\rho^{-2})^{1/2} - \rho'(1-Z_o^2\rho'^{-2})^{1/2} - Z_o(\phi+\phi') - Z_o\cos^{-1}(Z_o/\rho) + Z_o\cos^{-1}(Z_o/\rho')\right] .$$

To calculate the amplitude in (26) we determine θ and $\dot\theta$ from (22) and (38) and observe that $\mathcal{K} = 0$. First we find

(50) $$\dot\theta'\sec\theta'\tan\theta' = \dot\theta\sec\theta\tan\theta = -Z_o^{-1} .$$

$$(51)\quad \tan\theta+\dot{\theta}\sec\theta\tan\theta d(P,P_1) = \tan\left[\cos^{-1}(-Z_o\rho^{-1})-\phi\right] - Z_o^{-1}\rho\sin\phi\sec\left[\cos^{-1}(-Z_o\rho^{-1})-\phi\right]$$

$$= -(\rho^2 Z_o^{-2}-1)^{1/2}$$

$$(52)\qquad \tan\theta'+\dot{\theta}'\sec\theta'\tan\theta' d(Q,Q_1) = -(\rho'^2 Z_o^{-2}-1)^{1/2}$$

When (51), (52) and (49) are used in (26), the final result for u when $\rho\cos\phi > \rho'\cos\phi'$ becomes

$$(53)\qquad u(\rho,\phi) = (\rho^2 Z_o^{-2}-1)^{-1/4}(\rho'^2 Z_o^{-2}-2)^{-1/4}$$

$$\times \exp\left\{ik\left[\rho(1-Z_o^2\rho^{-2})^{1/2}-\rho'(1-Z_o^2\rho'^{-2})^{1/2}-Z_o(\phi+\phi')\right.\right.$$

$$\left.\left.-Z_o\cos^{-1}(Z_o/\rho)+Z_o\cos^{-1}(Z_o/\rho')\right]\right\} .$$

If $\rho\cos\phi < \rho'\cos\phi'$, $u(\rho,\phi)$ is obtained by interchanging primed and unprimed quantities in (53). In this case if P is near the tip, then $\rho << |Z_o|$ and (53) can be simplified. First we replace $\cos^{-1}(Z_o/\rho)$ by $i\log\left[\rho^{-1}(Z_o+\left[Z_o^2-\rho^2\right]^{1/2})\right]$. Then upon omitting some small terms, (53) becomes

$$(54)\quad u(\rho,\phi) = e^{-i\pi/4}(\rho'^2 Z_o^{-2}-1)^{-1/4}\ \exp\left\{ik\left[-iZ_o+\rho'(1-Z_o^2\rho'^{-2})^{1/2}-Z_o(\phi+\phi')\right.\right.$$

$$\left.\left.+Z_o\cos^{-1}(Z_o/\rho')+iZ_o\log(2Z_o/\rho)\right]\right\} .$$

If Q is also near the tip (54) can be further simplified.

Let us now compare our results (53) and (54) with the asymptotic form of the exact solution of the corresponding boundary value problem. We seek

a solution u of (19) with the source at (ρ', ϕ'). This solution must be defined in the region outside the wedge, i.e., in the region $0 \leq \phi \leq \alpha$. On the surface $\phi = 0$ it must satisfy (20) with Z given by (38). On $\phi = \alpha$ we require it to satisfy $u_\phi = 0$. The solution must also satisfy the radiation condition.

The solution of this problem has been found by L. Felsen [2]. It contains a surface wave given by

$$(55) \quad u(\rho,\phi) = -i\pi \frac{\cosh \eta(\alpha-\phi)\cosh \eta(\alpha-\phi')}{\alpha - i(kZ_o)^{-1}\sinh^2 \eta\alpha} J_{-i\eta}(k\rho_<)H^{(1)}_{-i\eta}(k\rho_>) \quad .$$

Here $\rho_<$ and $\rho_>$ denote respectively the smaller and larger of the two quantities ρ' and ρ. The constant η is a solution of the equation

$$(56) \qquad \coth \alpha = -\eta(kZ_o)^{-1} \quad .$$

For large k the solution of (56) becomes

$$(57) \qquad \eta \sim ikZ_o \quad .$$

Thus (55) becomes

$$(58) \qquad u(\rho,\phi) \sim \pi kZ_o\, e^{ikZ_o(\phi+\phi')} J_{kZ_o}(k\rho_<)H^{(1)}_{kZ_o}(k\rho_>) \quad .$$

We now insert the Debye asymptotic forms of the Bessel and Hankel functions for large k with order less than the argument in absolute value. Then (58) becomes, if $\rho > \rho' > |Z_o|$

$$(59) \qquad u(\rho,\phi) \sim Z_o(\rho'^2 - Z_o^2)^{-1/4}(\rho^2 - Z_o^2)^{-1/4}\, e^{-ikZ(\phi'+\phi)}$$

$$\times \exp\left\{ik\left[(\rho'^2 - Z_o^2)^{1/2} - (\rho^2 - Z_o^2)^{1/2} - Z_o\cos^{-1}(Z_o/\rho') + Z_o\cos^{-1}(Z_o/\rho)\right]\right\}.$$

This agrees exactly with (53). If $\rho < \rho'$ we must interchange ρ and ρ' in (59) and the result again agrees with (53) when it is similarly changed. If $\rho << |Z_o|$ and $\rho' > |Z_o|$ then (58) becomes identical with (54) when we insert the asymptotic form of the Bessel function for large k with order larger in magnitude than the argument, and omit some small terms. The agreement between the asymptotic form of the exact solution (55) and the solution (53) or (54) given by our method, is a confirmation of our theory.

8. Surface wave on a cylinder

Let us finally use our method to find the surface wave excited on a circular cylinder of radius a by a line source parallel to the cylinder axis. If the amplitude of the wave from the source is given by (24) then the surface wave is given by (26). We assume that the cylinder has a constant surface impedance. Then θ in (26) is a constant determined by (22). Let the polar coordinates of Q and P be $(\rho',0)$ and (ρ,ϕ) respectively. There are two rays from Q and P which traverse the cylinder in opposite directions. We first consider the shorter ray. See Figure 5. The two corresponding pairs of points Q_1, P_1 are found to have the coordinates $\rho(Q_1) = \rho(P_1) = a$ and

(60) $$\phi(Q_1) = \cos^{-1}(-Z)-\sin^{-1}\left(\frac{a}{\rho'}\sqrt{1-Z^2}\right) .$$

(61) $$\phi(P_1) = \phi-\cos^{-1}(-Z)+\sin^{-1}\left(\frac{a}{\rho}\sqrt{1-Z^2}\right)$$

Then we find that

(62) $$d(Q,Q_1) = \rho'\left[\sqrt{1-\left(\frac{a}{\rho'}\right)^2(1-Z^2)} + \left(\frac{a}{\rho'}\right)Z\right]$$

(63) $$d(P,P_1) = \rho\left[\sqrt{1-\left(\frac{a}{\rho}\right)^2(1-Z^2)} + \left(\frac{a}{\rho}\right)Z\right]$$

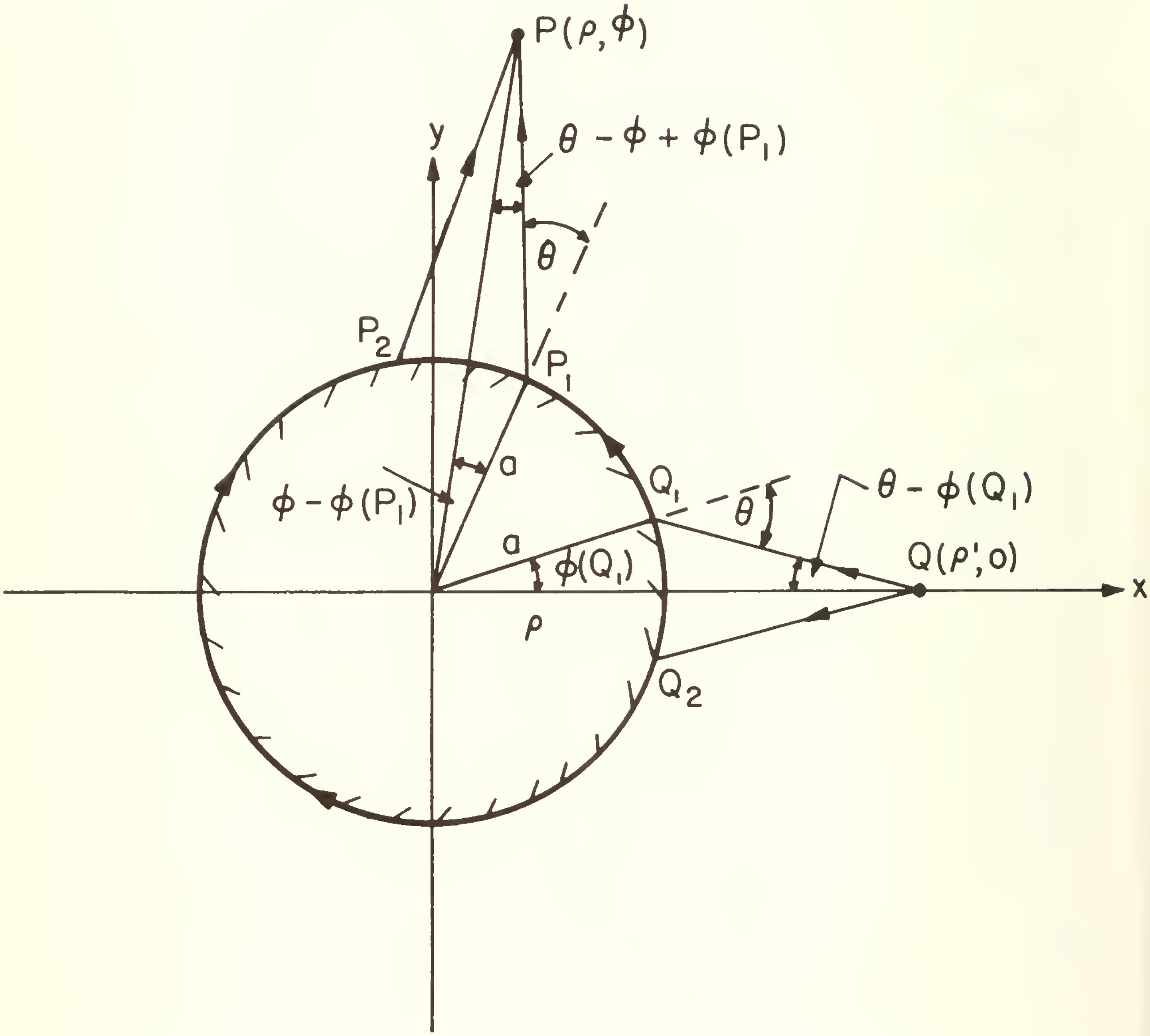

FIGURE 5

Excitation of a surface wave on a circular cylinder by a line source at Q. Two complex rays from Q hit the cylinder at the complex points Q_1 and Q_2 where they make the complex angle θ with the normal. This angle is determined by equation (22). The two complex surface rays which they excite travel around the cylinder in opposite directions. The complex space rays which they shed at P_1 and P_2 pass through P.

(64) $\phi_1(Q_1,P_1) = k_1 a\left[\phi(P_1)-\phi(Q_1)\right]$

$$= ka\sqrt{1-Z^2}\left[\phi-2\cos^{-1}(-Z)+\sin^{-1}\left(\frac{a}{\rho}\sqrt{1-Z^2}\right)+\sin-\left(\frac{a}{\rho'}\sqrt{1-Z^2}\right)\right].$$

Similar results hold for the longer ray except that ϕ is replaced by $2\pi-\phi$ in (61) and (64). Upon inserting these expressions into (26) and simplifying, we obtain

$$(65)\quad u(\rho,\phi) = \frac{-Z^2}{\sqrt{1-Z^2}}\left[\left(\frac{\rho}{a}\right)^2-(1-Z^2)\right]^{-1/4}\left[\left(\frac{\rho'}{a}\right)^2-(1-Z^2)\right]^{-1/4}$$

$$\times \exp ika\left[\frac{\rho}{a}\sqrt{1-\left(\frac{a}{\rho}\right)^2(1-Z^2)}+\frac{\rho'}{a}\sqrt{1-\left(\frac{a}{\rho'}\right)^2(1-Z^2)}\right.$$

$$+\sqrt{1-Z^2}\left\{\sin^{-1}\left(\frac{a}{\rho}\sqrt{1-Z^2}\right)+\sin^{-1}\left(\frac{a}{\rho'}\sqrt{1-Z^2}\right)\right\}$$

$$\left.+2Z-2\sqrt{1-Z^2}\cos^{-1}(-Z)\right]$$

$$\times\left[\exp ika\sqrt{1-Z^2}\,\phi+\exp ika\sqrt{1-Z^2}\,(2\pi-\phi)\right].$$

There are also other rays from Q to P each of which encircles the cylinder n times before leaving at P_1. The phase $\phi_1(Q_1,P_1)$ for such rays exceeds that given by (64) by the amount $2\pi nk_1a = 2\pi nka(1-Z^2)^{1/2}$. Their fields are computed from (65) by including this additional phase. When the fields on these rays are added together at P, a geometric series results. Its sum is the total surface wave at P which is

$$u(p,\emptyset) = \frac{-Z^2}{\sqrt{1-Z^2}}\left[\left(\frac{\rho}{a}\right)^2-(1-Z^2)\right]^{-1/4}\left[\left(\frac{\rho'}{a}\right)^2-(1-Z^2)\right]^{-1/4} \tag{66}$$

$$\times \exp ika\left[\frac{\rho}{a}\sqrt{1-\left(\frac{a}{\rho}\right)^2(1-Z^2)} + \frac{\rho'}{a}\sqrt{1-\left(\frac{a}{\rho'}\right)^2(1-Z^2)}\right.$$

$$+ \sqrt{1-Z^2}\left\{\sin^{-1}\left(\frac{a}{\rho}\sqrt{1-Z^2}\right)+\sin^{-1}\left(\frac{a}{\rho'}\sqrt{1-Z^2}\right)\right\}$$

$$\left. + 2Z - 2\sqrt{1-Z^2}\cos^{-1}(-Z)\right]$$

$$\times\left[\exp ika\sqrt{1-Z^2}\,\emptyset+\exp ika\sqrt{1-Z^2}\,(2\pi-\emptyset)\right]\frac{1}{1-\exp\left[2\pi ika\sqrt{1-Z^2}\right]}\,.$$

We shall now compare this result with the solution of the corresponding boundary value problem. That problem was formulated and solved in [3] for a cylinder with an impedance Z. However, Z was not assumed to satisfy (21) so that surface waves of the type we are considering were not found. Instead only the diffracted fields associated with the real surface rays were considered. Those fields were obtained by finding the solutions ν of the equation.

$$H_\nu^{(1)'}(ka) = -iZH_\nu^{(1)}(ka) \tag{67}$$

Each root ν yielded a wave given by [see reference [3], p 43, eq.(4)]

$$u = -\frac{i\pi}{4}\sum_m \frac{\cos\nu_m(\theta-\pi)}{\sin\nu_m\pi}\;\frac{\Omega H_{\nu_m}^{(2)}(ka)}{\frac{\partial}{\partial\nu}\Omega H_{\nu_m}^{(1)}(ka)}\;H_{\nu_m}^{(1)}(k\rho)H_{\nu_m}^{(1)}(k\rho') \tag{68}$$

where $\Omega\, F(x) = \frac{d}{dx}F(x)+iZF(x)$.

When Z satisfies (21) the equation (67) has an additional solution which corresponds to the surface wave. It can be found by using in (67) the Debye asymptotic formula for the Hankel function for order and argument both large and $|\nu| > ka$. Then (67) becomes

$$(69)\quad i\sqrt{\frac{2}{\pi}}\,\frac{1}{\left[\nu^2-(ka)^2\right]^{1/4}}\left\{\frac{\left[\nu^2-(ka)^2\right]^{1/2}}{ka}-iZ\right\}\exp\left\{-\left[\nu^2-(ka)^2\right]^{1/2}+\nu\cosh^{-1}\frac{\nu}{ka}\right\}=0\,.$$

The solution of (69) is

$$(70)\qquad \nu = ka(1-Z^2)^{1/2}\,.$$

When (70) is used in (68) and the asymptotic form of the Hankel function is employed, (68) becomes

$$(71)\quad u(\rho,\phi) = -\frac{Z^2}{\sqrt{1-Z^2}}\left[\left(\frac{\rho}{a}\right)^2-(1-Z^2)\right]^{-1/4}\left[\left(\frac{\rho'}{a}\right)^2-(1-Z^2)\right]^{-1/4}$$

$$\times \exp ika\left[\frac{\rho}{a}\sqrt{1-\left(\frac{a}{\rho}\right)^2(1-Z^2)}+\frac{\rho'}{a}\sqrt{1-\left(\frac{a}{\rho'}\right)^2(1-Z^2)}+\sqrt{1-Z^2}\left\{\sin^{-1}\left(\frac{a}{\rho}\sqrt{1-Z^2}\right)\right.\right.$$

$$\left.\left.+\sin^{-1}\left(\frac{a}{\rho'}\sqrt{1-Z^2}\right)\right\}+2Z-2\sqrt{1-Z^2}\left\{\frac{\pi}{2}-i\cosh^{-1}\sqrt{1-Z^2}\right\}\right]$$

$$\times\left[\exp ika\sqrt{1-Z^2}\,\phi+\exp ika\sqrt{1-Z^2}\,(2\pi-\phi)\right]\frac{1}{1-\exp\left[2\pi i ka\sqrt{1-Z^2}\right]}\,.$$

This is the asymptotic form of the exact surface wave (68). After some algebraic manipulations, it agrees exactly with the result (66) given by our method.

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